Early Math Resources for Teacher Educators

Does = Equal Makes? And What is the Magic Arrow?

Another Professor Ginsboo Story

by Herbert P. Ginsburg

Scene 1

One day, I, Professor Ginsboo, was deep in thought, strolling along a blank computer screen,

when a little girl emerged to face me. It was my dear little student, Menette!

"Professor Ginsboo, I have a problem. Yesterday in school, Ms. Zoller was teaching us about the = sign. She showed us this number sentence, which I depict in its entirety:

5 + 3 = ?

"She asked me, and I quote her, word for word, 'How much is five plus three?' Well, this was like easy-peasy because I have studied with the famous Professor Ginsboo."

I did not blush.

Menette continued. "I told her, and I quote, word for word, 'Five plus three makes eight. 'Well, you would have thought that I had called her some kind of lizard with tacky glasses. She got very upset, and said that the = sign does not mean *makes*. She said that I was being vulgar and should know that = means *equivalent to* because that is what she has been teaching me. But I don't understand why = does not mean *makes*, and I don't understand the meaning of *equivalent to*."

At this point, Menette and I were shocked to see a form emerging from behind a tree on the right hand side of the screen. Where did that tree come from? But it was now clear that the form was…

[Suspense]

Ms. Zoller, Menette's teacher!!!

She said, "I heard you complaining about me. These glasses are not tacky! And you are wrong. = does not mean *makes*."

Menette and Ms. Zoller then got into an argument. They said, and I quote, word for word:

M: "Yes it does."

Z: "No it doesn't."

M: "Yes it does."

Z: "No it doesn't."

Scene 2

At this point, I began to doubt both Ms. Zoller's teaching strategy and Menette's approach to studenting. So I separated the antagonists, stood in the middle, and began my lesson concerning operations and equivalence.

"First of all, you have to recognize that it's not enough to say, "Yes it does," or "No it doesn't."

I think they weren't listening because they said, and I quote, word for word:

M: “Yes it is.”

Z: “No it isn’t.”

M: “Yes it is.”

Z: “No it isn’t.”

I appealed for calm and proclaimed that they would enjoy my lesson on *makes* and *equivalence*. And given their tendency for contention, I unleashed on them my full arsenal of pedagogical techniques, my quiver of instructional arrows, my bag of teaching tricks, my garage of educational engines, etcetera, etcetera, etcetera!!!

“Menette,” I said, “I wish you to solve the following problem. Don’t write anything down. Just say your answer. How much is five and three?”

She answered, “Five and three makes eight.”

I wrote down, word for word:

Five and three makes eight.

Next I changed it a little:

5 + 3 makes 8.

I asked Menette if she thought that was right and if so why.

She said, “I am a very good informal adder (or maybe addist?). I have learned several strategies for figuring out addition problems.

“I can use blocks. I put five blocks here and three there and then I push them together and count them all to get the answer eight.

“Or I look at the five blocks, say *five*, and then count on to get eight.

“Sometimes I can do it in my head too. I have a picture of five blocks and a picture of three. Then I just count all of the blocks in my mind to get eight.

“Sometimes, I don’t use blocks or pictures in my head. I hear *five* and *three*, and then I just start with five and then count on three more to get eight.

“Sometimes, I can solve the problem by reasoning. I know that four and four makes eight. I know that five is just one more than four and three is just one less. So I change the five into a four and the three into another four, and then I know that the answer must be eight.

“I know how to do addition in many different ways. And whatever way I do it, in the end, five and three makes eight.”

I asked how she learned to do all this.

She responded, “I invented some methods myself. Also, sometimes people helped me to learn or improve the methods, and sometimes I embellished on their methods. I knew a lot before I even got to school. I didn’t need Ms. Zoller to teach me how to do what some scholars call *everyday math*. And I'm not the only one. Almost all of my friends can do the same thing.

"But it doesn't really matter how I learned the methods. It's very clear that when you add two numbers you get an answer, a result. So five plus three makes eight. = means *makes*."

Scene 3

I, Professor Ginsboo, was extremely proud of my student Menette's ability to articulate her mathematical knowledge. She understood a lot about addition. But was she right that = equals *makes*?

Now it was Ms. Zoller's turn to present her argument. She said, "The whole idea behind the number sentence 5 + 3 = 8 is to show that the quantity on one side is equivalent in number to the quantity on the other side. *5 + 3* is the same quantity as *8*.

"Look at it this way. Suppose you have five identical yellow blocks and three identical red blocks on one side of a balance, and you have eight identical green blocks on the other side. Every block, regardless of color, is the same weight. The two sides will balance because they are the same quantity. But if you have four yellow and three red blocks on one side and eight green ones on the other, the eight will plunge down to the ground, flipping the *4 + 3* all the way to Lewis Carroll's house in England. To avoid losing numbers in this manner, mathematicians have invented the concept of *equivalence*, which is symbolized by =. So = clearly means *is equivalent to*."

In response to this, I said, "Dear Ms. Zoller. I think that is a splendid and concise argument, well presented, with a dash of what I think you intended as humor. Ha, ha. And I'm beginning to admire your glasses too.

"But I believe that there is a flaw in your position, just as there is one in Menette's. Fortunately for you both, I will now clear up the contradictions, lift the fog of confusion, resolve yin and yang, and introduce a spirit of harmony to your relationship."

Scene 3 + 1 = 4:

(or maybe)

Scene 3 + 1 *makes* 4:

As you, dear reader, can see, this scene is so important that its heading contains two warring number sentences. Thanks for your silent applause.

I addressed both Menette and Ms. Zoller as follows:

"We begin with a number sentence like *6 + 3 = ?*. You both know that each numeral refers to a specific quantity. 6 refers to six units, and 3 to three units. The units can be anything: blocks, imaginary unicorns, or trains.

"You also know from the question mark that you have to find the sum of the two numbers. So as Menette said, this can be done in many ways, including counting objects, mental calculation, retrieving the sum from memory, and reasoning.

"Are you with me so far?"

Menette and Ms. Zoller nodded in appreciative agreement.

"So we can say that Menette is correct in maintaining that you have to get an answer—the correct sum. You might say that *6 + 3 = makes* nine, or that *6 + 3 = gives* nine. I prefer to say that **we** take *6 + 3 =* and *make* nine, but any of these formulations conveys the essential idea."

Menette was preening. Ms. Zoller looked extremely grouchy, even for a lizard.

I continued. "But there is more to this story. Suppose that you have six eggs (free range and organic) and three eggs on one plate. You figure out that there are nine eggs altogether, so you put nine eggs on another plate. Fine so far.

"But you also should understand that the six eggs and three eggs on one plate are equivalent in number to the nine eggs on the other plate. In one sense, you made something new when you put nine eggs on the second plate, but in another sense, the nine eggs are exactly the same in number as the six eggs and three eggs. *9* is equivalent to *6 + 3*."

Ms. Zoller was preening. Menette looked extremely grouchy, even for a young child.

"You should both be happy because you are both right. First you have to find out what the two numbers *make* and then once you have the answer, you need to understand that it is equivalent to the two numbers from which it was constructed.

"And I have to tell you that I, Professor Ginsboo, have devised a new symbol to clarify the roles of *makes* and *equivalence*."

Menette and Ms. Zoler each made one gasp, the sum of which was equivalent to two.

Scene 5

The = Sign and the Magic Arrow

I then unveiled the Magic Arrow.

"Here is a new way to think about addition. We begin with

$2 + 6 \Rightarrow ?$

"In this expression, the arrow tells us, 'Get the answer; find out what two plus six makes. What are you waiting for?'

"You then get the answer by whatever method you want. When you write down the correct answer, the $\Rightarrow$ changes in front of your eyes into =. So you begin with the new arrow symbol for the *operation* (adding), which then magically turns into the conventional equals sign to indicate the *relation* (equivalence).

"Also, suppose that the magic does not work (which might happen to you). You can just erase or color over the triangle at the end of the arrow, and then you will have =.

"Are you as impressed by my new symbol (and alternative magic) as you should be?"

(I didn't even have to look to know that they were.)

Dear reader, we can end the paper here for now because I have answered the original question concerning *makes* and *equivalence*. I recommend that you now interview a child so that you can achieve a deeper understanding of these issues. After that, you can return to the questions and answers presented in scene 6 (aka 3 + 3). And after that, I offer you scene 7, which introduces a new character, Julie the Teacher of Teaching. Of course, if you prefer, you can read on right now.

Scene 6

(aka 3 + 3)

Q & A

I then asked Menette and Ms. Zoller whether they had any questions. They did but insisted on anonymity, so I do not reveal the identity of the asker.

Q: "What if you respond to the arrow and add up two groups of eggs but get the wrong answer, without knowing you are wrong, and therefore don't put the right number on the second plate? How can you determine accuracy and fix the answer if it is wrong?"

A: "Ask each of the eggs on one plate to hold hands with one of the eggs on the other plate. If there are no extra eggs on either plate, then the numbers are equivalent. This is *one-to-one correspondence*. But if there are extras, scramble and eat them up, and then the numbers will be equivalent (although the extra eggs won't be happy)."

Q: "Does this work for subtraction too?"

A: "Like, duh!" (I can't believe I said that!)

Q: "What if you see this sentence?

$? = 4 + 2$

A: "Always point the arrow towards the question mark and proceed in the direction of the arrow, like this:

$? \Leftarrow 4 + 2$

Q: "What if you have this statement: *1 = 1*?"

A: "Well, it just says that one is equivalent to one. There was no need for the magic arrow because you don't have to make any number."

Q: "Why would anyone want to write *1 = 1*? Why is that interesting?"

A: “Isn’t it fascinating that one extremely large elephant is equivalent in number to one teeny ant?”

Q: “What if you have *2 + 3 = 1 + 4*? Are those sets of numbers equivalent?”

A: “You can’t tell unless you first determine the sum of the each set of two numbers. So do the following. First insert the magic arrow as follows:

“2 + 3 ⇒? Write the correct answer, *5*, whereupon the tip of the Magic Arrow disappears and you have *2 + 3 = 5*.

“Then repeat for the other numbers as follows:

“?⇐ 1 + 4, which eventually results in 5 = 1 + 4.

“Finally because you have determined that the two sets of numbers make the same answer, then they are equivalent in number: 5 = 5. But you have to figure out the answers to both statements (*2 + 3* and *1 + 4*) before you can decide whether they are equivalent. So *makes* comes before equivalence.”

Q: “What if you have 5 + _ = 6.”

A. “This is the missing addend problem, and it’s the only one that is a little tricky. What you have to do is consider that *five plus something* is equivalent to six. But you have to figure out how to make that something. You know that six is bigger than five, so you just change the problem to *6 – 5 ⇒?* and you are all set. Or you could try adding numbers to five and see what works to make six.

Well, at this point Menette and Ms. Zoller looked exhausted. I think they learned how interesting mathematical symbolism can be.

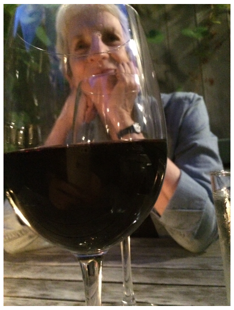
I, Professor Gisboo, know that you too have learned a great deal and I think that you will demonstrate your understanding by responding to this challenge:

Can you add five pieces of chocolate and three lions to make eight musical notes that are then equivalent in number to the chocolates and lions?

Scene 7

Julie the Teacher of Teaching

Although the computer screen was filling up with various characters and symbols, there appeared yet another person, attempting, for some reason I have never learned, to hide behind two wine glasses. She immediately proclaimed her identity (or at least part of it).



"Don't be alarmed. I am Julie the Teacher of Teaching. I will explain to you an excellent method for teaching *makes* and equivalence.

"First of all, begin with sentences like this: 5 = 5. You can show either the written numbers or five blocks on the right and five on the left (or both). Ask the children to determine whether these statements are true: Is five really equal in number to five? They can count the blocks; they can make a one-to-one correspondence between the two sets; and so on. They can take the five blocks on one side and break them up into a group of two and a group of three and determine that the number on that side altogether is still five. Ask them also to consider if cases like 5 = 6 are true, so that they learn to identify non-equivalent numbers.

"This method establishes from the outset that equivalence is the central mathematical idea to consider. Once this idea is clear, gradually expand the statement in both directions, asking the children to use blocks or mental counting to complete sentences such as:

$2 + 3 \Rightarrow ?$

$? \Leftarrow 1 + 4$,

$2 + 3 \Rightarrow \Leftarrow 1 + 4$

"In each case, once the child has determined the answer, return to the equivalence idea.

"You can even use this method to obtain the missing addend in

4 + _ = 5.

"Use blocks for this problem. Show the four blocks on one side and the five on the other, and ask what needs to be done to get the same number on both sides. To get the answer, children can experiment with placing different numbers of blocks on the left side after the set of four. If you do this with one as the missing number, they probably will see right away that a single block is required. Later, they can use the subtraction method described above ($5 – 4 \Rightarrow ?$).

Finally, I have to tell you that the first step in this process, teaching children about equivalence, is not going to be easy, as Piaget pointed out many years ago. So be patient and persist, and eventually almost all children will learn equivalence.

"So now that I have set you on the correct pedagogical path, we can raise one glass to *makes* and the other to equivalence. Bon mathématiques!"

And after this toast, she disappeared, as if in one gulp. Menette, Ms. Zoller and I were left to wonder why Julie the Teacher of Teaching was hiding and from whom. Maybe we will *make* the answer some day.