Professor Ginsboo Explains Everything You Need to Know about

Pattern and Algebraic Thinking

and/or

How to Get Un-bored (with a song) by A B A B A B and Accomplices

By Professor Ginsboo

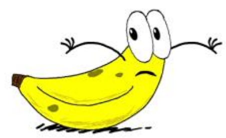
Setting the Stage

One day, I, Professor Ginsboo, was deep in thought, strolling along a blank computer screen,

when a little girl emerged to face me. It was my dear little student, Menette,

along with her teacher, Mz. Zoller (whose reptilian appearance should in no way detract from her developmentally appropriate pedagogical prowess).

Before, I, Professor Ginsboo, could say a word, a motorcycle and a banana raced across the screen,

magically dropping hundreds of As and Bs from the motorcycle’s exhaust. The screen, usually pristine (except for a few fingerprints) was covered with so many

A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A Bs

that it was impossible to see me, Professor Ginsboo, or Menette and Mz. Zoller on the screen underneath them. Naturally, I said the secret words for poofing the cheeky A B A B A B A B A B A B A B scoundrels, and of course the inundation went poof!

I am going to spare you the details, but the motorcycle and the banana then returned with tons of Green Red Green Reds, not to speak of buckets of Blue Yellow Blue Yellows, but each time they crowded the screen I poofed the intruders.

Menette and Mz. Zoller were, of course, enormously appreciative and praised my praise-worthy poofing.

Menette said, “This intrusion of the A Bs and the Green Reds and the Blue Yellows takes place almost every day at school. All we have been doing for months (well, it seems that long) is study A B patterns and Green Red patterns and Blue Yellow patterns. This makes me very upset.”

Mz. Zoller wailed that all she did for months was *teach* A B patterns and Green Red patterns and Blue Yellow patterns and she was at least as disturbed as Menette.

Menette and Mz. Zoller then sang, *a capella,* in unison, and on key (more or less):

There was a teacher had a girl,
And bored-o was her mind-o
B–O–R–E–D
B–O–R–E–D
B–O–R–E–D
And bored-o was her mind-o.

There was a teacher had a girl,
And bored-o was her mind-o
(clap) –O–R–E–D
(clap) –O–R–E–D

(clap) –O–R–E–D
And bored-o was her mind-o.

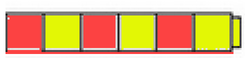
There was a teacher had a girl,
And bored-o was her mind-o
(clap)–(clap)– R–E–D
(clap)–(clap)– R–E–D
(clap)–(clap)– R–E–D
And bored-o was her mind-o.

“Oh, dear,” I exclaimed. “You can stop here—I know what’s coming. Do not be disturbed and distressed. We will confront the dastardly motorcycle and banana and thwart their evil A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B A B plot.

“But first we must understand our LETTER enemy. Let me tell you about A B A B and Green Red Green Red and Blue Yellow Blue Yellow.”

A B A B and the Others Too

I commenced by showing Menette and Mz. Zoller the following interlocking cubes. In the dialogue that follows I will refer to myself as Professor Ginsboo and to Menette and Mz. Zoller as M and Z respectively. On reflection, to maintain parity I will abbreviate my name as well.



Professo Ginsbo: What do you see here?

M: I see a red cube, a yellow cube, a red cube, a yellow cube, a red cube, and a yellow cube.

Professo Ginsbo: Mz. Z, what do you see?

Z: I see a pattern.

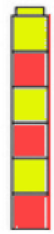
Professo Ginsbo: Can you tell me more about the pattern?

Z: It’s a repeating pattern. It starts on the left, and red and yellow is the basic pattern unit, which is repeated twice. Then the pattern units can repeat indefinitely.

Professo Ginsbo: Can the pattern start on the right and repeat ad infinitum to the left?

Z: Of course.

Professo Ginsbo: What if you stood up the pattern like this?



Z: That's OK. You can add onto the top or the bottom.

Professo Ginsbo: Let's review what I so effectively taught you.

First, you *perceived* some objects. You saw them as red and yellow cubes. In other scenarios, you could have touched, without looking, a rough object and a smooth object, and so on. You could have heard a loud sound and a soft sound, and so on. You could have tasted a sweet morsel and a bitter morsel, and so on. You could have felt your arm being raised up and moved down, and so on. You could have smelled a pleasant odor or a noxious one. You begin with the evidence of your senses—what you see, touch, hear, taste, smell, and feel within your body. Did I miss any of the senses? Does all this make sense?

M and Z: Brilliant, just brilliant.

Professo Ginsbo: Of course. But there is more.

Second, you notice something larger than the individual elements you have just seen, touched, and so on, namely a pattern.

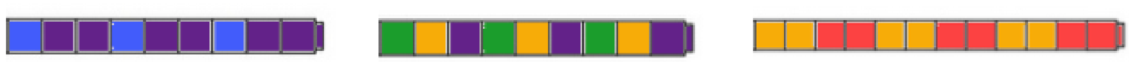
M and Z: Like duh!

Professo Ginsbo: Never mind "Like duh!" After identifying that the objects constitute a pattern, you then explain *why* it is a pattern, *what makes it* a pattern, what *structure underlies* the pattern, and what *rule* can be used to create the pattern and describe it. And once you know this, you can extend the pattern as long as you want.

M: So, once we *perceive* the elements and *identify* the whole as a pattern, we *analyze* the pattern so that we can extend it. That's very clear. Not like duh! But still, it's not all that interesting.

I, Professor Ginsboo, was then startled when the motorcycle and the banana raced across the screen, this time scattering the patterns below.

Professo Ginsbo: What do you make of these?



M: Easy peasy! In all three, the pattern unit is larger than the A B pattern unit that bores us so much. Here, the pattern units are blue–purple–purple, green–orange–purple, and orange–orange–red–red. After you know the pattern unit, you just repeat and repeat and repeat. Still so boring!

Z: She is right: just more of the same. So unfair! So boring!

There was a teacher had a girl,
And bored-o was her mind-o
B–O–R–E–D
B–O–R–E–D
B–O–R–E–D
And bored-o was her mind-o.

Professo Ginsbo: Stop, stop! You are now ready for three much more interesting patterns.

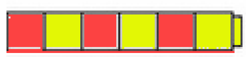
At this point Menette raised her hand, jumped up and down, and uttered squeals.

M: Wait. Before we go on, I want to raise two important, disturbing, and deep questions.

Menette's Two Questions

Question 1

M: First, look at this again.



It's a pattern because it alternates A and B. There are two separate pattern units: A, and B. The pattern starts with an A, then has a B, and then an A, then a B, and so on. It could end in A or B, because you could just continue alternating to extend further.

Z: No, that's wrong. It has to be a repeating pattern with red–yellow as the basic unit.

M: No it doesn't.

Z: Yes it does.

M: No it doesn't.

Z: Yes it does.

M: No it doesn't.

Z: Yes it does.

Professo Ginsbo: You can stop now. Both of you are making sophisticated arguments in support of your positions. Let's leave it for our readers to settle: can the pattern end on A (which in this case is *red*) or must it end on B (which in this case is *yellow*)? Another way of putting it is: to B or not to B?

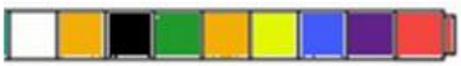
Question 2

M: Here’s my second question. Look at this. Is it a pattern?

Z: Obviously not. The last cube ought to be a red, because the pattern unit is red–green.

M: But look at this. It is a pattern. The pattern unit is red–green–red–green–red–green–green, and it repeats, and can go on endlessly.

Now take a look at this. Is this a pattern?

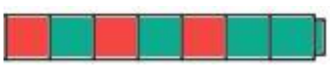


Z: Obviously not. It’s just a random mix of colors!

M: But look at this. The pattern unit is the whole first array and it repeats, and can go on endlessly. Why is it not a pattern?

Menette then proclaimed that she could prove her contention in an algebraic fashion. Here is what she said.

M: Let’s look again at the first unusual pattern I showed you.



At first it looks wrong, but then, when you see this, you know that it is indeed a splendid pattern unit, repeated three times.

So here is my proof.

- Let’s call the first pattern W, for (apparently) wrong.
- Add another W. Then another W.
- Doing so is the first step in repeating and extending the pattern unit W indefinitely.
- Q. E. D. (Abbreviation for the Latin, “I nailed it.”)

Or you might say that three Wrongs make a Right!

Z: That's ridiculous and absurd. It's not fair. You could do this for anything.

Professo Ginsbo: It appears that you could. What do you think, dear reader? No doubt you will easily determine whether Menette's logic makes sense.

M and Z: Can we go back to school now?

Prof G: Surely you jest. There is so much for you to learn! Here is something really interesting—a new kind of pattern!

Growing Patterns and Algebraic Thinking (Do Not be Alarmed)

Having convinced them to remain, I, Professor Ginsboo, addressed my visitors again. "Dear Menette and Mz. Zoller, I, Professor Ginsboo, will now introduce you to a new pattern. We will begin by analyzing your song. Please don't sing it. Just look at the written version."

There was a teacher had a girl,
And bored-o was her mind-o
B–O–R–E–D
B–O–R–E–D
B–O–R–E–D
And bored-o was her mind-o.

There was a teacher had a girl,
And bored-o was her mind-o
(clap) –O–R–E–D
(clap) –O–R–E–D
(clap) –O–R–E–D
And bored-o was her mind-o.

There was a teacher had a girl,
And bored-o was her mind-o
(clap)–(clap)– R–E–D
(clap)–(clap)– R–E–D
(clap)–(clap)– R–E–D
And bored-o was her mind-o.

"That's enough. Let's use our ingenious method for thinking about patterns. First, what do you see?"

Menette said that she sees words and letters.

"Excellent," I said. "That is what you see. And of course had you sung the song, you would have heard the words and the names of the letters."

Menette then said that she sees a pattern too, although it is much more complicated than A B A B.

Mz. Zoller interrupted to say with considerable excitement, “The first step in our system was to *perceive* the elements. We did that. We saw the words and letters. The next step is to see if I can *identify* a pattern! There is definitely a pattern here. And I can do the third step, too, namely *analyzing* the pattern.”

I said, “Ah, dear Mz. Zoller, you are so thoughtful. Tell us your analysis. Please explain *why* the song is a pattern.”

“There is definitely repetition at the beginning and at the end of each verse. At the beginning is:

There was a teacher had a girl,
And bored-o was her mind-o

And at the end of each verse is

And bored-o was her mind-o.

“You might think of both the beginning and end of each verse as pattern units that can be repeated and extended indefinitely.”

Applause!!

Menette then said, “Your idea of repeating pattern units is insightful. But I noticed a pattern within each verse.

“In the first verse, we see:

B–O–R–E–D
B–O–R–E–D
B–O–R–E–D

“But then we see the following in the second and third verses:

(clap) –O–R–E–D
(clap) –O–R–E–D
(clap) –O–R–E–D

(clap)–(clap)– R–E–D
(clap)–(clap)– R–E–D
(clap)–(clap)– R–E–D

“And here is the pattern. The first verse includes only letters, five of them, and zero claps. The second verse has one clap and four letters. The third verse has two claps and three letters. This is a *growing pattern* in which there are 0 claps in verse 1, after which there is 1 clap in verse 2, and then there are 2 claps in verse 3.”

Splendid! But next I wanted to see how deeply Menette understood the pattern, so I asked her how many claps there would be in verse 5.

She decided to use a table to help her figure out the answer. Here it is.

Verse Number (V)	Number of Claps (C)
Verse 1	0
Verse 2	1
Verse 3	2
Verse 4	?
Verse 5	?
Verse 6	?

I asked what she could learn from this table about number of claps in verses 4 and 5. Here is what she said.

"I know that there must be 3 claps in verse 4, and 4 claps in verse 5. And I will tell you how I know. I looked at the column of claps, and saw that there is a series of numbers going down the column, namely 0, 1, and 2. And after that, I reasoned, the series would continue with 3, 4, and 5. So the answer to your question is that there would be 4 claps in verse 5."

Four times as much applause as in the previous instance!!!!

Menette continued: "But that is only one method I used, and not even the more interesting. You will notice that for verses 1, 2, and 3, the number of claps is always one less than the number of the verse. By this method I figured out there must be 4 claps in verse 5.

"Furthermore, I wanted to get a general solution, so I wrote this down.

> *Number of verse is always one more than number of claps.*
>
> $V = C + 1$
>
> *Or, the number of claps is always one less than the number of the verse.*
>
> $C = V - 1$

"So," Menette continued, "I did a little algebra to solve the problem!"

Applause!!!!!

I, Professor Ginsboo, was enormously impressed with Menette's method.

But Mz. Zoller interrupted, saying, "That is excellent, Menette. But there is another way to think about this problem. It's really a shrinking pattern. The number of letters decreases in each new verse. So there are 5 letters in verse 1, 4 letters in verse 2, and so on. Look at this table.

Verse Number (V)	Number of Claps (C)	Number of Letters (L)
Verse 1	0	5
Verse 2	1	4
Verse 3	2	3
Verse 4	?	?
Verse 5	?	?
Verse 6	?	?

"So Menette had a growing pattern in which the number of claps increases by 1, and I have a shrinking pattern in which the number of letters decreases by 1. My pattern is the opposite of hers."

I must admit that I, Professor Ginsboo was greatly impressed by Menette's and Mz. Zoller's observation of elements, identification of pattern, and use of the tables to analyze the pattern.

Now, dear reader, think about how delicious the patterns are. At the outset and the end of each verse we have simple repetition of these pattern units:

At the beginning of each verse is:

There was a teacher had a girl,
And bored-o was her mind-o

And at the end of each verse is:

And bored-o was her mind-o.

As these pattern units repeat endlessly, a growing pattern and a shrinking pattern operate simultaneously, with one the opposite (inverse) of the other.

So scrumptious! Oh, the joys of mathematics. I swoon!

Menette and Mz. Zoller allowed that this lesson was definitely more interesting than the one before. But then, unaccountably they said, "Let's go home."

I replied, "No, not now. I have one more important lesson for you!"

Spatial Relations and Symmetry

“I, Professor Ginsboo, will now take you into the real world so that you can examine extraordinary and pleasing patterns. Never mind A B A B or even the song *Bored-o*. We will encounter marvels beyond belief.”

Menette and Mz. Zoller asked in unison, “What is the real world?”

I told them that it exists outside of the computer screen on which we spend almost all of our lives (except during sleep mode). I told them not to be frightened, but to follow me, and hold hands during the journey.

“Look at this. It’s called a *sidewalk*, and it exists in *Prague*, located in the real world. First, what do you notice? After that you will tell me whether it is pattern, and if so, what makes it a pattern.”

Menette responded, this time in bullet format (as only a computer screen resident can):

- First, I see lots of more or less square tiles, and some rectangles.
- Second, it is a pattern, and a beautiful one too!
- Third, it’s a pattern because the individual tiles are grouped into larger squares or rectangles. Look at the pattern unit in the middle.

- Dark tiles form four rectangular borders, and within the borders four squares are placed at the tips (vertices) of a larger square.
- Further, the pattern unit repeats in both directions along the sidewalk.

"Dear Menette," I exclaimed loudly, "you have produced an excellent analysis of the sidewalk pattern, which does indeed extend in both directions (although not indefinitely, or even to New York). No doubt you have been inspired by my earlier, excellent lesson on shapes. *[See Professor Ginsboo: Shapes in the Spatial Relations Module].* Let me show you another sidewalk picture and then a picture of a floor from another real world place called *Vienna*. You can analyze them for fun."

As soon as the floor picture appeared, Mz. Zoller began crying. I expressed sympathy but could not empathize. “Why in the world are you crying? There is nothing here that requires such a response. Suck it up.”

Mz. Z instead explained. “I am weeping tears of pedagogical joy. What a floor! It has so many beautiful shapes of different kinds. It is not limited to common triangles, squares and rectangles, but also includes some seldom seen polygons. Some shapes are far away and others close. Some extend to the left and some to the right. Some are within others. Some appear to be on top of others. Like Menette’s sidewalk, this floor affords analysis of shape and spatial relations.

“Moreover the floor provides wonderful opportunities to examine symmetry! Imagine a vertical line drawn through the middle of this part of the picture. Then hold up a two-sided mirror on that vertical line. The mirror image on the left side looks like the side on the right and the mirror image on the right side looks like the side on the left.

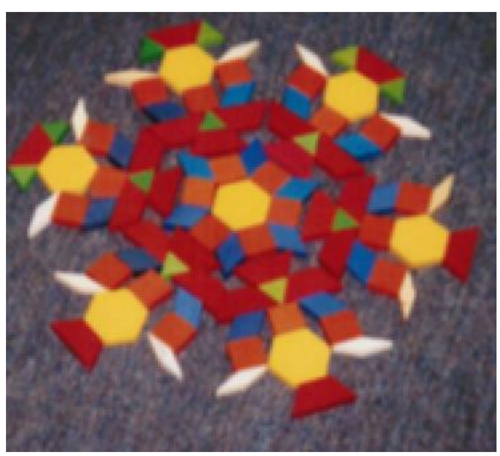
“The two sides are symmetric! They reflect, as in our imaginary mirror. This is another kind of pattern.”

She wanted to swoon, but reptiles can't.

Menette took advantage of the swoon-time to say that there were so many reflection symmetries on which to reflect. "Look at my new dress (which I lent to a hanger).

"Look closely to see divine reflection symmetries and spatial arrangements!"

Then Mz. Zoller emerged from her unsuccessful swoon state, and exclaimed, "This suggests an excellent pedagogical method that I will call the *Zoller APProach*, or *ZAPP*. It involves the following. Give children some pattern blocks. They will then create something like this.



"Analyze and talk about the structure's spatial and symmetric qualities. You can also do this with ordinary blocks."

The real world had made Menette and Mz. Zoller extremely happy.

Lessons learned

Menette then interjected, "Let me summarize Professor Ginsboo's important teaching.

- First, we see some elements, identify a pattern, and then analyze it to figure out why it is a pattern. These three steps are also followed in one form or another in science as well as by detectives attempting to solve a crime.
- Second, there are several kinds of patterns, among them: repeating, growing, decreasing, and symmetric.
- Third, my symmetric dress is gorgeous.

Are these lessons correct?"

Mz. Zoller and I, Professor Ginsboo, both exclaimed in unison, "Yes, they are correct, but obviously almost all the credit should go to your teachers. Of course, if you were not correct, that would be another story entirely."

Mz. Zoller added the following:

"Now I can engage my students in so many interesting and challenging pattern activities! My profuse gratitude to you, Professor Ginsboo."

This of course is the perfect note on which to conclude my pattern lesson.