5 Ways to Improve Children's Understanding of Length Measu

Mi Yeon Lee and Dionne Cross Francis

Measurement comprises an important set of concepts that are fundamental for students to competently navigate and work with objects and structures within their physical environment. Despite their importance, these concepts are difficult for students to grasp, as evidenced by low performance on measurement items on the National Assessment of Education Progress (NAEP) (Battista 2007, 2012; Kamii 2006). In particular, students seem to have difficulty understanding the concept of length (Battista 2006; Kloosterman 2006). In this article, we describe five activities to support elementary school teachers in building students' conceptions of measurement. We begin by briefly reviewing the relevant Common Core State Standards for Mathematics (CCSSM) and key concepts essential for understanding length.

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These activities can support elementary school teachers in building students' conceptions of measurement.

Measurement standards

In the early grades, CCSSM for measurement include two main aspects: (1) describing and comparing *measurable* attributes and (2) classifying objects and *counting* the number of objects in each category. The first aspect entails two standards:

(a) Describe *measurable* attributes of objects, such as length or weight, and describe several measurable attributes of a single object (K.MD.A.1, CCSSI 2010, p. 12).

(b) Directly compare two objects with a *measureable* attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference (K.MD.A.2, CCSSI 2010, p. 12).

Another key standard in first grade includes being able to order three objects by length and compare the lengths of two objects indirectly by using a third object (1.MD.A.1, CCSSI 2010, p. 16). Distinguishing between "measureable" and "countable" is important for understanding measurement. Both measuring and counting are quantitative representations of objects. To count, students need to know (1) the sequence of counting words, (2) how to match the number sequence in a one-to-one correspondence with the objects in the set being counted, and (3) that the last word in the counting sequence indicates the amount of the set. However, to measure something, we need to consider something else—*a unit* and *the iteration of the unit*. More specifically, to count, we focus on the number of discrete objects in a set. However, to measure, we usually compare an attribute of an item (e.g., length) with a unit that has the same attribute. That is, *measuring* can be defined as an activity to compare an attribute of an object being measured with the same attribute of a given unit by iterating the unit (Sarama and Clements 2009). Although measuring and counting are different operations, because measuring does involve counting—to measure an attribute of the object, we count the number of units we are iterating—distinguishing between them may be difficult. The teachers in our professional development sessions seemed to find this distinction challenging.

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Concepts required to understand length measurement

According to Sarama and Clements (2009), at least eight conceptual understandings are required to build strong understanding of length:

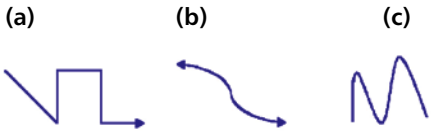
- Attribute
- Conservation
- Transitivity
- Equal partitioning
- Units and unit iteration
- Accumulation of distance and additivity
- Origin
- Relationship between number and measurement

Attribute relates to understanding that lengths span fixed distances. *Conservation* involves understanding that the length of an object does not change when the object is moved. *Transitivity* includes understanding that if the length of object A is equal to the length of object B, and the length of object B is equal to the length of object C, the length of object A is equal to the length of object C.

Equal partitioning refers to a mental action of dividing an object into equal parts or units. *Units* and *unit iteration* are required to understand the operation of placing the length of a small unit repeatedly along the length of the object being measured. *Accumulation of distance* entails understanding that the number words signify the space covered when counting all iterations of a unit. *Additivity* involves understanding that length can be decomposed and composed. *Origin* includes understanding that any point on a ratio scale can be used as the origin. *Relation between number and measurement* includes understanding that counting

FIGURE 1

Before starting the Three Paths task, the authors used masking tape to create three large, nonstraight paths on the floor. They asked teachers to determine which path would be longer, first estimating and then measuring the three paths with a chosen nonstandard unit.



Adapted from Van de Walle, Karp, and Bay-Williams's (2013) Crooked Paths activity

discrete objects can be a basis for measuring. Below we describe the activities that we designed to help elementary school teachers involved in a five-day professional development workshop integrate these eight core measurement concepts into lessons on length.

Expanding understanding of length measurement

The main ideas in measurement progress from recognizing and comparing lengths, to measuring with nonstandard units, to using measuring tools based on standard units. Thus, the activities we describe are organized in an instructional sequence recommended for helping students develop a conceptual understanding of measurement. The goal was to not only extend the teachers' knowledge of measurement but also help them be adequately prepared to explore ideas about length with their students.

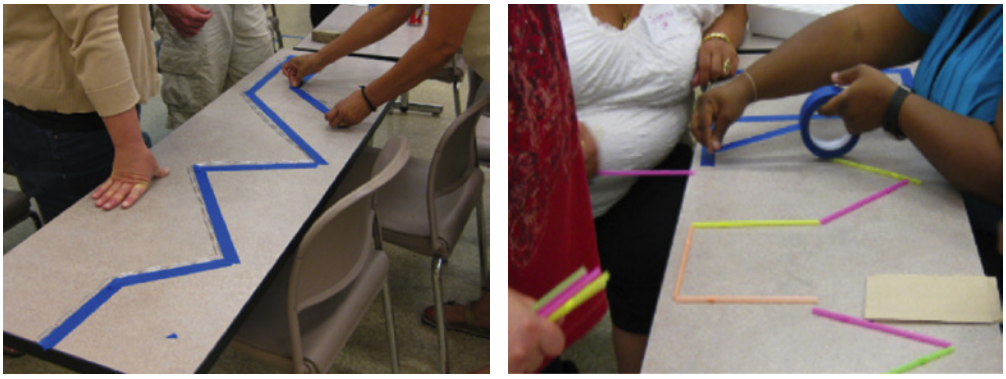
Activities 1 and 2: Defining length

The first activity was to find three or four objects in the room; measure them with an item such as a straw, toothpick, or paper clip; then record the measurements (e.g., The pair of scissors is less than three toothpicks long; my pencil is one paper clip longer than my crayon). Teachers identified that this activity helps teach the concepts of attribute, unit iteration, and conservation. They also recognized that measuring an object by iterating a nonstandard unit (i.e., the length of the paper clip or toothpick) helps teach that the length spans a fixed distance and does not change, although the object is moved to compare the lengths of different objects.

The second activity was the Three Paths task. Before starting this activity, we created three large nonstraight paths on the floor using masking tape (see **fig. 1**). Then we asked teachers to determine which path would be longer, first estimating and then measuring the three paths with a chosen nonstandard unit. After measuring the three paths, the teachers were expected to compare their lengths and put them in order. Finally, teachers were asked to create another path that would be as long as path c (see **fig. 2**).

FIGURE 2

After measuring the three paths, the teachers compared and ordered their lengths. Then they created other nonstraight paths that would be as long as path c.



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Through engagement and discussion, teachers identified the activity as a good one for exploring several of the core measurement concepts:

- The *attribute* of length can be addressed by conceptualizing that length on a nonstraight path is not simply the distance between two endpoints; thus, crooked and compacted paths may be longer than spread-out paths.
- By measuring three paths with nonstandard units rather than stretching them out side-by-side, the concept of *transitivity* among the three paths (e.g., path c is longer than path b; path a is longer than path c; therefore, path a is the longest) can be covered.
- In selecting a tool and measuring the length by iterating the selected tool, the concepts of *units* and *unit iteration* as well as the ability to select an appropriate nonstandard unit to measure an object can be taught.

Activities 3 and 4: Using literature to support understanding of length

We used children's mathematics literature to ground the third and fourth activities. Incorporating literature is useful for supporting students' understanding of measurement because many stories involve the use of nonstandard units and serve as rich resources for connecting measurement concepts to children's experiences (Clarke 2001; Cross et al. 2012).

We began the third activity by reading the book, *How Big Is a Foot?* (Myller 1991). In this story, a king asks an apprentice to make a bed for the queen. To determine the size of the bed, the king uses his footprint as the measuring unit. But when making the bed, the apprentice uses his own footprint, which is smaller than the king's. After reading the story, the following question was posed: "Why was the bed too small for the queen?"

To answer the question, we encouraged the teachers to use their footprints to measure the chalkboard in the room. Each teacher traced a foot on paper, cut it out, and used it to measure the length of the blackboard. Then they listed the measurements, in footprints, by teacher (e.g., 10 Sara feet, 8.5 Jordan feet, 11.25 Mary feet). Teachers identified multiple ways that the activity could support students' understanding of length and measurement more broadly. In particular, the activity targets understanding the concepts of *attribute of length* and *unit iteration*. In addition, by relating the size of the footprint to the number of footprints to measure the board, three key ideas become visible:

1. The larger the unit, the smaller the measure.
2. Lengths are not always whole numbers.
3. Standard measurement units are necessary for effective communication.

For the fourth activity, we began with reading, *How Tall, How Short, How Far Away?* (Adler 2000). This book introduces the ancient Egyptian units of a *digit* (the width of one finger), a *palm* (the width of four fingers), a *span* (the distance from the tip of the thumb to the end of the little finger), and a *cubit* (the distance from the elbow to the tip of the middle finger). Then teachers were asked to—

- use strings to measure their height in cubits, spans, palms, and digits;
- find the relationships among their cubits, spans, palms, and digits; and
- express their heights in different ways (e.g., 3 cubit = 2 cubits, 3 spans = 9 spans = 2 cubits, 5 palms).

Teachers identified that this activity can be used to support students' understanding of the concepts of *attribute*, *transitivity*, *units*, and *unit iteration,* as well as help them consider why using standard units is important. To guide their thinking in this regard, teachers suggested asking such questions as, "Can we

FIGURE 3

Transitioning to standard units of measurement, for the fifth activity, the teachers made rulers with two different colors of strips.

Adapted from Van de Walle, Karp, and Bay-Williams's (2013) Make Your Own Ruler activity

figure out who is the tallest in our classroom by comparing our height measures? Why or why not?" Teachers also suggested extending this activity to include additional concepts, such as *unit conversion* and *place value*, by having students experience trading a cubit for some number of spans, palms, or digits.

Activity 5: Transitioning to standard units of measurement

The fifth activity was to make rulers with two different colors of paper strips (see **fig. 3**):

1. Make several strips with two different colors for assembly.
2. Connect the strips end to end in alternating colors.
3. Use strip rulers to measure objects in the classroom.
4. After using the strip ruler, number the strips progressively, and then compare the strip ruler with a standard ruler.

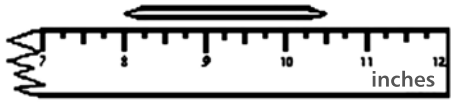
Teachers identified that this activity includes several concepts required for understanding length measurement: *equal partitioning*, *relation between number and measurement*, *accumulation*, *additivity*, and *origin*. To make their own strip ruler, students have to equally partition two different sets of colored strips into unit pieces. To measure with the ruler, students have to count the number of strips spanning the length of an object, which can lead to understanding accumulation of distance and additivity. Finally, by putting numbers on the strip ruler and comparing the strip ruler with a standard-unit ruler, students can understand the importance of origin in measuring length. In particular, such questions as the following are useful for teachers to ask when helping students build the meaning of origin:

- What do the numbers on the standard ruler mean?
- What is the starting point on each ruler?

In addition, teachers identified that this activity can be helpful to prevent or remediate misconceptions related to the use of a standard-unit ruler, such as reading the number aligned with one end of the object—when it is placed midway long the edge of the ruler—as the measurement, without considering the alignment of the other end of the object. In the 2003 NAEP (see **fig. 4**), only 20 percent of fourth graders could give the correct measure of an object that was not aligned with the end of a ruler (Kloosterman 2006). By engaging students in this activity, teachers can help them focus on aligning the intervals on the standard-unit ruler rather than the hash marks or numbers (Bamberger, Oberdorf, and Schultz-Ferrell 2010).

FIGURE 4

On this NAEP test item for grade 4, only 20 percent of students could give the correct measure of an object that was not aligned with the end of a ruler (Kloosterman 2006).



Adapted from the National Assessment of Educational Progress (NAEP) 2003 Mathematics Assessment

Focusing on length measurement

During the year since the professional development (PD) workshop, teachers have observed some of the challenges students face when engaging in the five activities. For example, in activity 1, students might have difficulty selecting the right size of measurement tools and using them correctly. In activity 2, students could have difficulty identifying the longest path when curvy and straight lines are given together, by counting the number of parts in each line without considering variable lengths of the parts. In activity 3, students might include gaps between their footprints rather than fitting them end-to-end when they measure the blackboard. Also, because their footprints have five different sizes of toes, students might be confused about the point from which to measure.

In one PD participant's kindergarten classroom, although the students experienced similar difficulties with activity 3, even these young students were able to construct a key measurement idea—that the larger the unit, the smaller the measure. Thus, although the activity might be challenging for some teachers and young students, it can be a worthwhile

experience for students to develop a strong concept of length measurement.

Drawing on the work of Battista (2007), we know that many students encounter challenges in maintaining the connection between numeric measurements and the process of unit-measure iteration. Thus, students should first be engaged in activities that help them understand measurable attributes and that an object's measurement is the number of iterations of a nonstandard unit that constitutes an attribute (e.g., activities 1–4). Experiences using nonstandard units are important in the initial stages of learning length measurement because they help students focus on the length attribute rather than procedures of measurement (Battista 2012). Without this initial focus, students could have difficulty developing deep understandings of measurement concepts. The five activities described above are offered as useful resources for improving students' understanding of length measurement.

Common Core Connections

1.MD.A.1
1.MD.A.2

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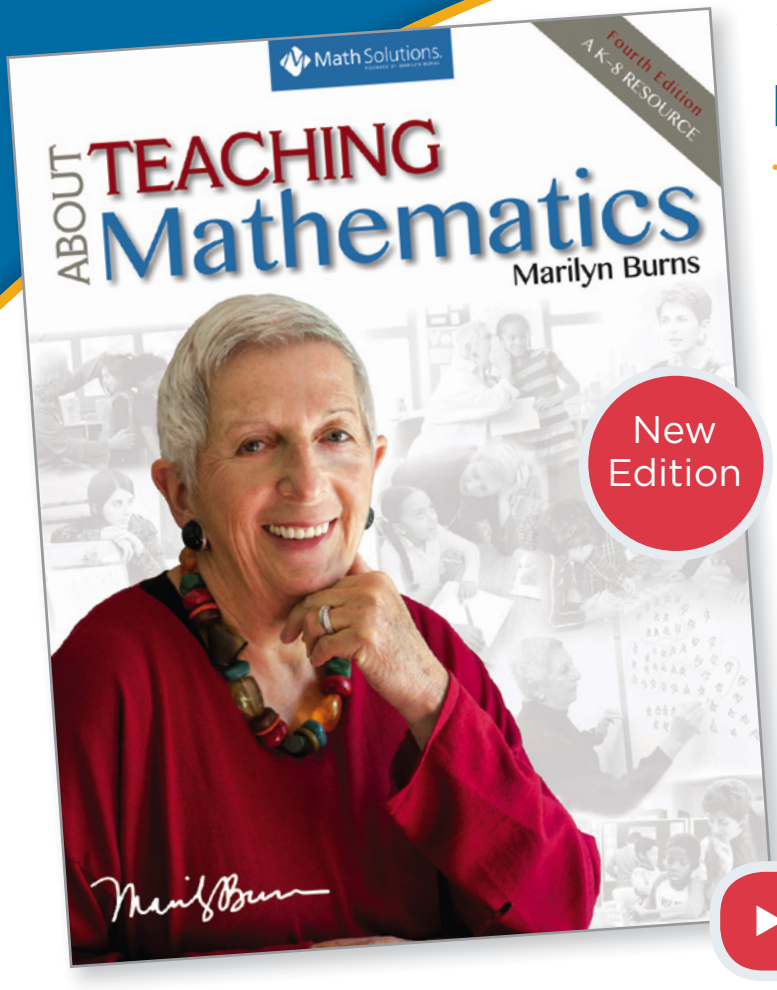
We can't teach what we don't know, and we can't teach well what we don't understand.

– Marilyn Burns



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