Math Matters:

Children's Mathematical Journeys Start Early

Report of a Conference held November 7 and 8, 2011
Berkeley, California

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Overview

The conference "Pathways for Supporting Early Mathematics Learning" was motivated by a growing understanding of the importance of early mathematics in the lives of children (ages 3 through 8 years) and by major changes in the national educational landscape. Held in Berkeley, California, the conference brought together researchers, curriculum developers, educators involved in teacher preparation, and policy makers to forge an agenda for progress on the overarching question: ***What must be done to ensure that early childhood mathematics experiences are meaningfully linked to a K-3 system based on the Common Core State Standards?***[1] Given that the issues faced in understanding students' mathematical growth and the potential pathways to support that growth are the same across the nation, the conference had a national focus. It was held with the objective of creating agendas in the areas of research, educational practice, and policy/advocacy.

What the Research Says: Early Math Skills Matter

Mounting evidence indicates that the mathematics knowledge children develop before entering elementary school is critical to later academic achievement. In a widely cited study of longitudinal data, Duncan et al. (2007) report that in a comparison of math, literacy, and social-emotional development at kindergarten entry, "early math concepts such as knowledge of numbers and ordinality were the most powerful predictors of later learning." These findings are echoed in studies by Romano et al. (2010) and Grissmer (2011). Indeed, research consistently indicates that early mathematical proficiency is associated with later proficiency not only in mathematics, but in reading as well (see Volume 6, Issue 5, of *Developmental Psychology*) and may even be linked to rates of high school graduation. Although the mechanisms underlying such associations are not yet understood, the importance of early math — and thus of access to it for all students — is clear.

Moreover, children who begin school with poor math skills typically do not catch up. As shown in the figure on the next page[2], across the nation, children who have low math scores in the fall of their kindergarten year continue to lag behind their better-prepared peers through the 8th grade. Those least prepared are disproportionately children of color and from low-income families. Clearly, any serious effort to close the achievement gap needs to include, if not focus on, children before school entry.

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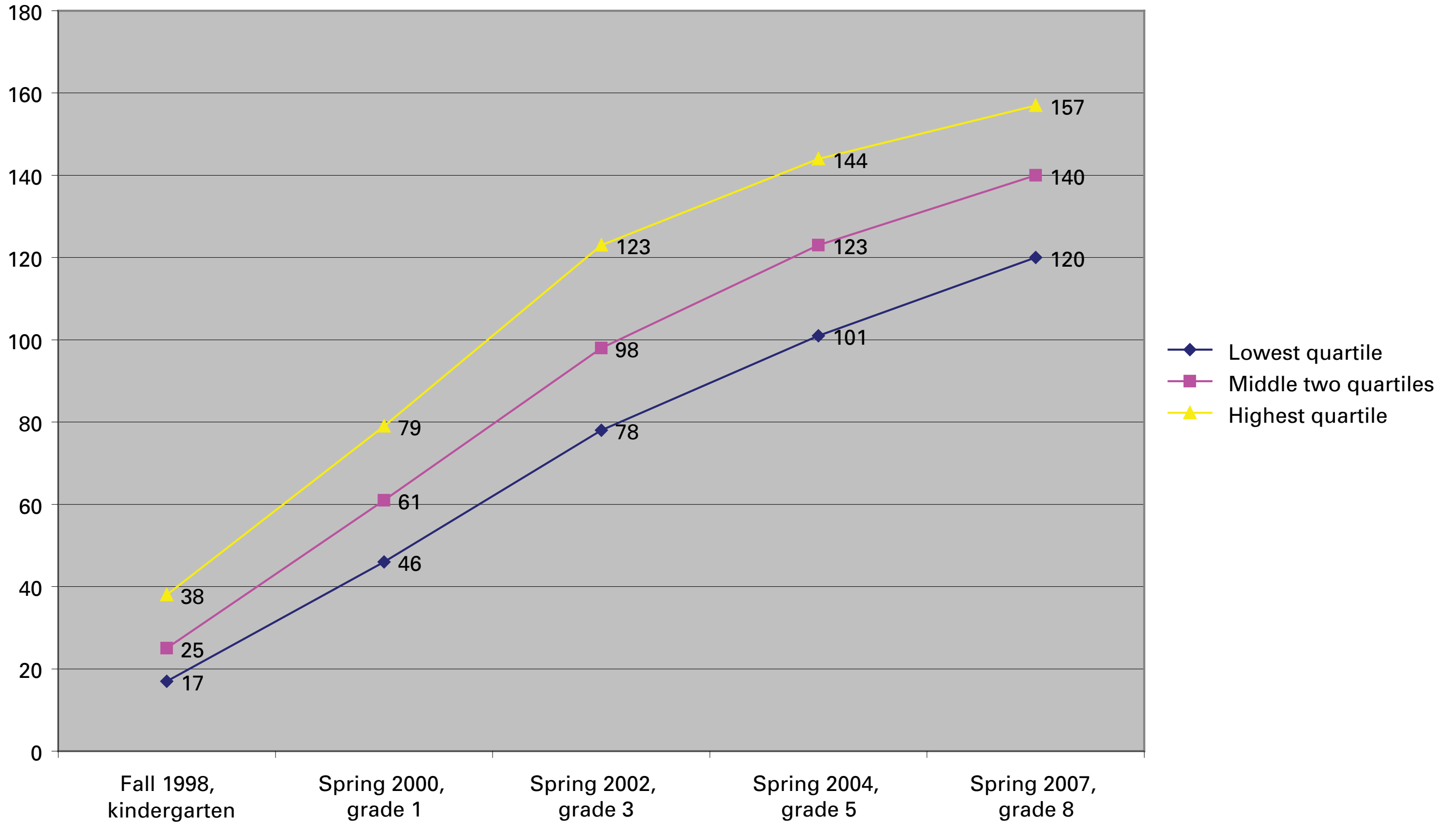
1 The conference was sponsored and funded by the Heising-Simons Foundation. The perspectives presented in this report are those of the conference participants.

2 Resource: National Center for Education Statistics, Early Childhood Longitudinal Study, Kindergarten Class of 1998–99, fall 1998, and spring 2000, 2002, 2004, and 2007; and National Science Foundation, Division of Science Resources Statistics.

Mathematics Achievement Scores of Kindergartners Followed Through Grade 8, by Kindergarten Score Quartile



Math achievement (kindergarten through 8th grade: SOURCES: National Center for Education Statistics, Early Childhood Longitudinal Study, Kindergarten Class of 1998–99, fall 1998, and spring 2000, 2002, 2004, and 2007; and National Science Foundation, Division of Science Resources Statistics.

Common Core Has Implications for Early Math

The *Common Core State Standards in Mathematics* (CCSSM, 2010) have now been adopted by 46 states as the backbone of their mathematics instruction. The new Common Core will result in greater homogeneity in the early school mathematics experiences of students across the United States. It will also compel states to offer richer math instruction than is required by most of the previous state standards. The Common Core standards focus on grades K-12, and do not address the preschool years. However, their broad acceptance implies that there will be increased breadth, depth, and coherence in the early elementary grades. This, in turn, has important implications for early childhood education (ECE), whether it is delivered in home- or center-based child care or preschool programs.

While teacher education programs vary across states, overall there is a disjunction between the preparation of ECE and elementary school teachers. Children's experiences at both levels vary greatly. The 2009 National Research Council (NRC) report, *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, proposed a set of recommendations for moving toward research-based common goals and practices in ECE. The NRC report predated the K-12 Common Core. The Common Core's widespread adoption offers an opportunity and an incentive to update and refine the findings of the NRC report, and to build a pragmatic agenda to enhance children's early mathematical experiences and prepare them for what they will encounter in school. The Berkeley conference was organized with this opportunity in mind. Conferees met in four sessions:

- *Session 1: Research.* What is known about young children's mathematical competencies, how can they be supported, and how do they affect later academic achievement?
- *Session 2: Educational Practice: Curriculum and Pedagogy.* What are the qualities of effective math instruction and curricula, as well as practices in the home, for diverse groups of young children (including English-language learners), and how do we assess effective instruction?
- *Session 3: Educational Practice: Professional Development and Articulation.* What kind of teacher preparation and support is needed to promote well-articulated and effective mathematics instruction for young children, from early childhood through grade three?
- *Session 4: Policy and Advocacy.* What state and federal policies are needed with regard to standards, assessment, and teacher/child development specialist credentialing? How can these policies be effectively promoted?

This report first details the main points of consensus from each of the sessions. It then summarizes recommendations made by conference participants.

Session 1:

Research: Supporting the Development of Children's Math Skills

What is known about young children's mathematical competencies?
How can they be supported?
How do they affect later academic achievement?

Early Math Myths

Myth #1: Early math is all about numbers.

Myth #2: Children must reach a certain level of cognitive maturity before they can "think mathematically."

Myth #3: Problem-solving skills come *after* learning the basics, rather than serving as a *way* to learn mathematics.

Early mathematical proficiency predicts not only later mathematical success but also success in other domains. Moreover, a great deal is known about how to promote mathematical development in children. The following vignettes illustrate some of what is currently known about young children's mathematical abilities and how they can be enhanced.

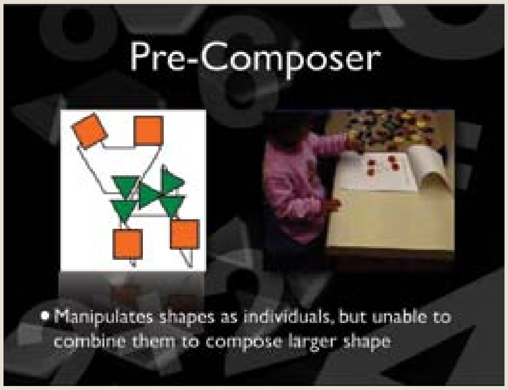
Vignette A:

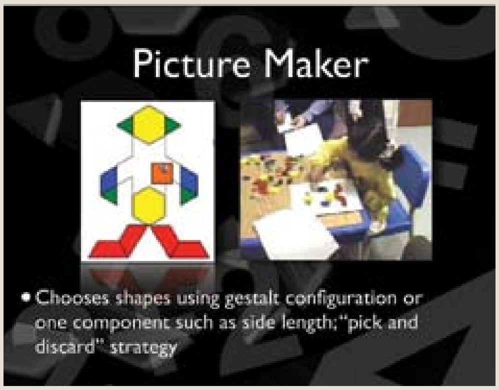
Understanding the concept of number involves more than counting.

The teacher puts three dolls on the table and asks 3-year-old Ben what will happen if she gives him two more. He says there will be five. When she gives him the two additional dolls he puts them on the table, confident that there are five. He can count them in any order. Ben *knows* that there will be five dolls altogether, even with two out of sight and regardless of how they are arranged on the table. His classmate, Jolene, is unsure; she places the two new dolls alongside the three, and counts from the beginning: "One, two, three… [pause]… four, five." She has to count them in view and lined up to answer the question. Ben and Jolene both answered correctly, and both can count to five. But they represent two different stages of understanding.

Vignette B:

Geometry is about more than naming.





Courtesy of Douglas Clements, University of Buffalo, SUNY

Four-year-olds Maria and Charlene work on a task that requires them to place different geometric shapes atop sketches of the pictures. Maria places some of the shapes atop the figure, but one by one, without matching the features of the figure; the result is the figure at the left. Charlene notes the shapes she needs — the green pieces "fit" the sides of the head, the red pieces "fit" the feet and legs, and so on. She comments on the properties of the geometric objects — for example, a square *couldn't* be part of the legs, because the angles of the square don't match the angles of the legs. Early introduction to *geometry* is about more than naming shapes. It includes seeing how they "fit" in various ways; it includes perceiving and representing patterns. Maria and Charlene are developmentally at different points with regard to the geometric objects, and they would profit from different kinds of support.

These two vignettes demonstrate that assessing children's mathematical understandings to guide the next steps in their instruction requires careful attention. They also suggest that providing children with opportunities to play, explore, and encounter mathematical patterns and structures as they engage with mathematics in various ways will help them to develop foundational mathematical understandings (Ginsburg, Inoue & Seo, 1999; Sarama & Clements, 2009).

The vignettes illustrate how children may come to conceptual understandings of mathematics. Even in an example as simple as counting (Vignette A), the idea is not to focus on the rote production of a numerical sequence, but rather to develop key understandings, for example, that no matter how one divides up a set of five objects (3 and 2, 2 and 3, 1 and 4, 4 and 1) the sum is always five; that addition and subtraction are related in important ways, etc. Similarly, geometry (Vignette B) is not simply about naming or defining shapes, but about seeing how pieces fit together, understanding that one can compose larger shapes from smaller shapes and decompose larger shapes into smaller shapes. If there is one theme that cuts across all of mathematics, it is that mathematics itself fits together, and understanding mathematics is a process of sense-making — of seeing how mathematical ideas cohere. Mathematics provides a language for "capturing" and discussing patterns in numbers and space. It can be seen as the codification of patterns and intuitions, and thus as "natural" in important ways. Mathematics instruction that consists of rote memorization divorced from sense-making is problematic (Schoenfeld, 2012).

The Common Core standards define mathematical activity and understanding broadly, focusing both on mathematical content and mathematical practices. The CCSSM[3] states that students should engage in the following mathematical *practices*:

- making sense of problems and persevering in solving them;
- reasoning abstractly and quantitatively;
- constructing viable arguments;
- modeling with mathematics;
- using appropriate tools strategically;
- attending to precision;
- looking for and making use of structure;
- looking for and expressing regularity in repeated reasoning

Such practices are just as appropriate for preschool-aged children as they are for older children. Anyone who has seen a young child persevere at playing tic-tac-toe (which is a fundamentally mathematical game), making or copying a building or other structure out of blocks, or dividing a pile of plastic cookies while playing house, is watching the development and use of mathematical practices. The Common Core standards reflect what we know about how children should learn math.

Children need to have opportunities to grow mathematically and be supported in that growth.

Rich mathematical activities are engaging. They meet children where they are and offer opportunities and structures for the development of deeper understandings. Rich learning environments provide feedback and scaffolding. In Vignette B, for example, the student can see whether a piece fits by trying to fit it to the picture. But a teacher's or caregiver's question, "What kind of piece do you think might go there?" can help the child develop planning strategies as well as focus on particular features of the geometric objects. Similarly, asking children "If you put the X there, what do you think I'll do?" during a game of tic-tac-toe helps them develop strategic, logical thinking — mathematical thinking. Children's responses provide important information to the teacher about what they understand and what kind of "scaffolding" (extra support, guidance, and experiences — not necessarily "telling") they need to master a mathematical concept, and what kinds of challenges they might profitably be given next.

3 http://www.corestandards.org

Children need linguistically rich and culturally meaningful mathematical activities.

Learning mathematics can and should support the development of literacy, and vice versa. The activities described above are language rich. Communicating effectively in and with mathematics (e.g., explaining why a certain method for sharing cookies is fair, or why you don't want to make a particular tic-tac-toe move) contributes both to deepening mathematical understanding and to developing linguistic fluency. Mathematical activities also need to be culturally responsive, to build on the knowledge and experience that students bring with them to all formal and informal learning environments (Civil & Khan, 2001; González, Andrade, Civil, & Moll, 2001). A dress-maker showing how she cuts patterns for clothing, or children making "ojos de dios" or copying the geometric patterns in an ornamental rug links mathematics to children's lives, and makes their mathematical experiences more personally relevant[4].

Further research is required on the needs of low-income children.

Ginsburg notes, "In general, we know a great deal about the development of mathematical thinking, as described in several comprehensive accounts (Baroody, Lai & Mix, 2006; Clements & Sarama, 2007; Ginsburg, Cannon, Eisenband & Pappas, 2006; Nunes & Bryant, 1996)… [but we] need to learn a great deal more about the competence and learning potential of 'disadvantaged' children — that is poor, minority children growing up with limited resources in difficult neighborhoods. The research on these children yields results that are far more complex than usually assumed, showing a mix of competencies and lack of them (Ginsburg & Russell, 1981)".

What children learn is a function of their *opportunities to learn*. The challenge for research is to identify points of strength and points of leverage that can be used to help the nation's children develop to their full potential. The challenge for policy makers is to find ways to provide adequate resources and effective policies for the next generations of Americans.

Further research is needed related to developmental progressions.

Developmental progressions — the sequences in which most young children learn mathematical concepts — provide some basis for developing ECE learning standards that are linked to learning standards for K-3. Conference participants recommended caution, however, because developmental progressions are not well understood. Some topics obviously precede others on mathematical grounds, but others may be subject to experience, so an individual's progression is likely to be a complex mix of content-related hierarchy and that individual's experience. Mathematics progressions should not be conceived of as linear pathways through hierarchically-ordered collections of skills. Learning in any domain is multi-dimensional (see, for example, the above description of the complexity of ostensibly simple domains like counting and elementary geometry), and children should be given opportunities to grow along various dimensions simultaneously. Following a rigid sequence in teaching is therefore not recommended. What is critical is for teachers to know what children understand and what they don't understand. This entails frequent assessments of children's mathematical thinking. This kind of ongoing "formative" assessment can be used to guide the selection of activities that support the development of deeper mathematical understandings.

A series of documents produced by the National Council of Teachers of Mathematics, beginning with *Curriculum Focal Points for Pre-K through Grade 8 Mathematics* (2006) and including various support materials (e.g., Fuson, Clements & Beckman, 2010), can serve as a useful guide for developmentally appropriate mathematics for young children.

4 See, for example, www.brooklynkids.org/attachments/MexicanFolkArt_HiRes.pdf

Session 2:
Curriculum and Pedagogy

What are the qualities of effective math instruction and curricula, and good practices in the home, for diverse groups of young children (including English-language learners), and how do we assess effective instruction?

Myth #4: Mathematics learning emerges naturally if kids are engaged in rich activities.

Myth #5: Mathematics learning should follow one clear, prescribed path.

Myth #6: All math learning for young children needs to be in the context of play.

Effective Math Instruction

Vignette C:
Mrs. Rivas asks her first graders to make up a word problem for 6 = 2 + ☐. The following conversation takes place at a table:

Maria: We could do a story about lions.

Oscar: Okay… we have six lions and two lions. How many does that make? That's easy, (counting up from six on his fingers) six, seven, eight! The answer is eight!!!

Mrs. Rivas: (approaching the table): What's your story?

Oscar: There were six lions and then there were two more lions.

Mrs. Rivas: Mike, I see you've copied the problem down. Can you read the problem out loud?

Mike: (reading from the paper) Six equals two plus a box.

Mrs. Rivas: So what is the box for?

Heejung: We put the answer in it — so six equals two plus the answer.

Oscar: But the number sentence is wrong, the equals can't go at the beginning.

Mrs. Rivas: What does the equals sign mean?

Maria: Everything on one side of the equals has to equal everything on the other side.

Mrs. Rivas: Is there another way to talk about the problem?

Oscar: Six lions equals two lions plus how many lions?

Maria: How 'bout there are six lions in the pride and two lions are in the tree, so how many lions are under the tree?

Mrs. Rivas: You all are really working on this! Who can tell me how many lions are under the tree?

Oscar: Four! Because two lions and four lions makes six lions! The number that goes in the box is four!

Mrs. Rivas: Mike, can you write the number sentence for us? (Mike writes 6 = 2 + 4)

Mrs. Rivas: Is that number sentence true?

Heejong: Yeah!

Mrs. Rivas: How do you know?

Heejong: Because everything on one side of the equals is the same as everything on the other side, and there is a six on one side and, because two plus four is six, there is six on the other side, so six equals six.

This exchange between this group of children and their teacher illustrates the kind of mathematical conversations that young children are capable of having and that help them develop a deep understanding of mathematical concepts, in this case equality. Note that the teacher's goal is not for the children simply to come up with the right answer, but rather to engage in active mathematical problem solving.

Dale Farran reported that in her observations of pre-kindergarten and Head Start classes, increasing the amount of children's math talk from 2% to only 4% of the day led to significant math gains (Hofer, Cummings & Farran, under review). In addition to developing their own mathematical understandings, the more children talk, the more they give teachers information about what they understand and how they are reasoning. Conversations about math are also helpful in developing language skills for all children, including English-language learners for whom practice in conversation is especially important.

Consider also the tasks discussed in Vignette B, in the previous section. These are two in a series of increasingly complex geometric tasks, in which young children are given the opportunity to make pictures composed of geometric shapes. This activity incorporates many of what conference participants viewed as important components of instructional tasks given to young children:

- Children have concrete materials to manipulate (although concrete materials are not always needed).
- The task is engaging for young children.
- The task involves an important mathematical concept (that shapes can be decomposed and composed to make other shapes).
- Children are engaged in a process of problem solving that requires some effort and persistence.
- The task is amenable to children working collaboratively and discussing alternative solutions.
- Embedded in the task is ongoing feedback to inform children's actions.
- The completed task provides an opportunity for children to observe and take pride in the fruits of their problem-solving activity.

It is essential to evaluate instruction in terms of the opportunities children have to learn mathematics, not just in terms of what the teacher does. The goal is to help children *develop, discuss, and use efficient, accurate, and generalizable methods to solve mathematical problems.* To achieve this goal, young children need to be given latitude to construct their own strategies for solving problems.

Conference participants discouraged the overuse of some activities commonly seen in early childhood classrooms. Learning to count by rote teaches children number words and their order, but it does not teach them number sense. Knowing that four follows three is of minimal value if a child doesn't know what four means. Marking the day on a calendar introduces children to the concept of days of the week, weeks, and months, but it does not teach math. Coloring and pasting macaroni in squares on paper with numbers on them is an extremely inefficient way to teach math. Paper-and-pencil tasks (e.g., drawing a line from the numeral 4 to a picture of four apples; coloring in an outline of the numeral 4) are fine for practice, but they don't teach children one-to-one correspondence. Teaching math effectively requires a focus on children's understanding of the core foundational concepts in mathematics. Consider, for example, the interactions below.

THOMAS PERKINS/PHOTOS.COM

Building on What Children Know and Care about in Informal Activities

Vignette D:

In the play area in a preschool, three children are having a loud discussion over how to share Duplo train parts: fourteen cars, three smokestacks, and two cow catchers. They are a hot commodity in the block area and are a regular cause for unhappiness for children who do not get a turn to build a train complete with smokestack and cow catcher. As the teacher approaches, the children are arguing about who gets to have the first turn with the train. As is evident by the numbers, there are not equal parts for everyone.

Susan: Teacher, they aren't being fair!

Teacher: Okay, let's talk about how we can share the trains.

Susan: First, Jonas can have a turn, and then I can have a turn, and then Sunaina can have a turn.

Teacher: Is that okay with everyone?

Sunaina: No! I don't wanna be last, it'll be too late!

Teacher: Is there another way we can share the trains?

Susan: We could have little trains. Then everyone could have a train.

Jonas: There isn't enough cow catchers.

Teacher: Should we count the cars, smokestacks, and cow catchers and see if we can share them somehow?

Children: Okay.

Teacher: Let's count the cars. (They all count together… "one, two…"). Hmmm… let's see if we can share them so that everyone has the same. Jonas, can you give everyone the same number of trains? (Jonas tries, but they don't create equal sets for each child.)

Susan: (She looks at the trains in front of everyone.) How about we each have four and then we put two in the middle?

Teacher: Okay — can you do that Jonas? (Jonas gives each child four trains and then puts two in the middle.) Okay, let's work on the smokestacks next. Sunaina, can you give everyone the same number of smokestacks? (Sunaina gives each child one.)

Sunaina: We each get one!

Teacher: Okay, what do we do with the cow catchers?

Children: (They answer in chorus.) I want one!

Teacher: Susan, how many cow catchers do we have?

Susan: One, two. Two!

Teacher: Jonas, do we have more children or more cow catchers?

Jonas: Children. How 'bout if I get one and Sunaina gets one and then we take turns? Sunaina could give hers to Susan when she gets done.

Teacher: Who would get a longer turn then, you or Sunaina?

Sunaina: I don't want a short turn!

Susan: I know, we could give the two extra trains to the person who doesn't have a cow catcher and then we'll all take turns!

Teacher: Is that okay with everyone?

Children: (They answer in chorus.) Uh huh.

In this interaction, the teacher takes advantage of the children's engagement in a task to conduct an impromptu math lesson on equivalencies and social problem solving. The vignette illustrates that play and instruction do not necessarily compete for time, but can be effectively integrated via interactions that simultaneously build oral language, social-emotional development, and mathematical skills.

Children profit both from math play and from structured curriculum.

Mathematics concepts may be learned and conveyed through activities that children experience as play — but mathematics learning does not *automatically* happen through play. Play or games can effectively reinforce and expand upon what children learn during more focused instructional times (see, e.g., Ginsburg, Lee & Boyd, 2008; Klibanoff, Levine, Huttenlocher, Vasilyeva & Hedges, 2006).

Some evidence suggests that well-prepared teachers who have a clear sense of the mathematical goals for their students and are sensitive to young children's mathematical thinking do not need to employ a structured curriculum in the early primary grades (Carpenter, Ansell, Franke, Fennema & Weisbeck, 1993). Such preparation is rare among elementary school teachers, and the training required is even less likely to occur at scale for teachers of pre-kindergarten children. There is evidence that specific mathematics curricula can produce gains (Clements & Sarama, 2008; Preschool Curriculum Evaluation Research Consortium, 2008), and a well-planned curriculum, informed by what is known about children's mathematics developmental progressions, is recommended for teachers who do not have extensive expertise in the teaching of mathematics.

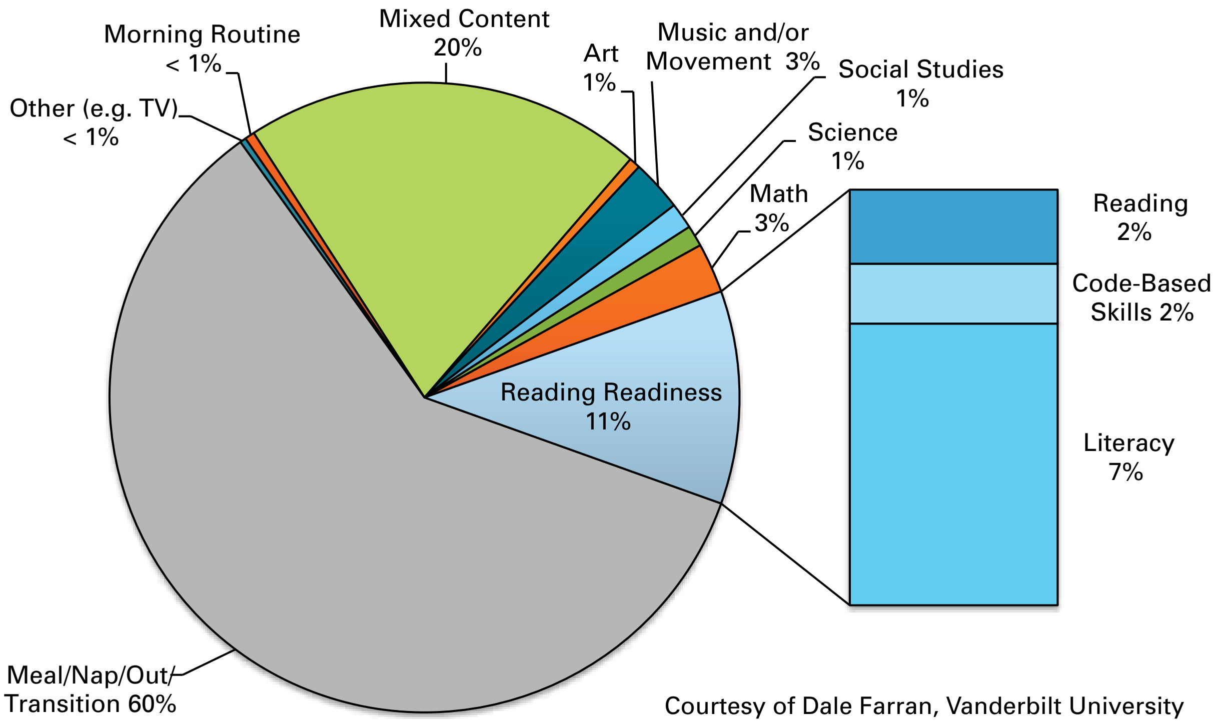
A curriculum should provide scaffolds and supports for teachers to question students and assess their understandings, give guidance for next steps based on those assessments, and build opportunities to learn about children's thinking on a daily basis. It should not, however, be followed rigidly. Teachers should adapt the curriculum to the needs of *their* students by checking children's understanding continually, and employing different teaching strategies when the one being used is not effective.

Children need focused time for mathematics instruction.

Time needs to be set aside for intentional instruction that has structure, clear math learning goals and that is sensitive to the students' current understanding. Dale Farran and others have found a strong association between the amount of teacher-led math instruction and gains children make in mathematics (Farran, Lipsey & Wilson, 2011).

Research suggests that the amount of time currently devoted to mathematics instruction for young children is far from adequate. Dale Farran reported that in her study of early childhood classrooms, math was intentionally taught by teachers about 3% of the day while literacy was taught 11% of the day (see pie chart below, Farran et al., 2011). Others have found similar amounts and similar imbalances between a focus on literacy and on math (e.g., Winton & Buyssse, 2005). Teachers are often concerned that increasing time for math will reduce children's opportunities to develop literacy skills. But studies indicate that increasing math instructional time can both increase math learning and promote language and literacy skills. Learning is not a zero-sum game in that mathematics activities that call for discussion and explanations contribute to both mathematical understandings and to literacy. In previous research, a half hour per day of focused math instruction and activities has shown substantial benefits for children's math skill development. In the absence of clear research evidence on the optimal amount of time for math instruction, participants agreed that 30 minutes per day is a reasonable guideline, although they pointed out that this focused time should be supplemented with math games, integrating math into play and other activities, and taking advantage of informal "teachable moments" throughout the day.

Proportion of Time Spent in Preschool Content Areas



Courtesy of Dale Farran, Vanderbilt University

Children need to engage in meaningful math.

Teachers need to consider children's identities as learners and performers of mathematics as well as their mathematical skills and understanding. Conference participants emphasize, again, that it is important to make sure that children are given opportunities to see how mathematics applies to their everyday experiences, such as a lesson in measurement in the context of baking muffins or having children count pennies to buy things in a play store. At the meeting, Carpenter discussed the "mathematization" of children's experience to describe what teachers can do — redescribing, reorganizing, abstracting, generalizing, reflecting on, and giving language to what children first understand on an intuitive, informal level. Participants also recommended adapting culturally familiar activities that children do in the home (e.g., playing cards, games) for use in the classroom.

Session 3:

Professional Development and Articulation

What kind of teacher preparation and support are needed to promote well-articulated and effective mathematics instruction for young children, from preschool through grade three?

Standards and preparation for mathematics teaching vary widely across the U.S., although there will be much greater consistency in K-3 once the Common Core standards gain traction.

The goals for ECE programs and K-3 are typically viewed differently. For ECE, goals include caring for children and preparing children socially and emotionally, as well as academically, for elementary school. Publicly-subsidized state preschool and the federal Head Start program also aim to prepare children from low-income families to succeed in school as part of the war on poverty and efforts to close the achievement gap. In contrast, the goals of the early elementary grades are primarily academic. The public also has different views concerning the value of ECE and K-3 education. The status of ECE as worthy of public investment is debated, whereas K-3 education is seen as a necessary and public good.

The two sectors also differ in the importance conferred upon mathematics. Math is considered a core domain of instruction in the elementary grades. In ECE, social-emotional development has historically been seen as the primary goal. Over the past decade, language and literacy development have been given increased prominence and are now widely considered important. Mathematics in ECE settings varies from "none" to "some," however, and even when it is present, it is typically given relatively superficial attention. Moreover, it is important not to merely increase the *quantity* of mathematics taught in ECE, but also to increase the quality. Research has demonstrated that increasing one does not necessarily increase the other (Varol, Farran, Bilbrey, Vorhaus & Hofer, in press).

The preparation of ECE educators varies across states more than the preparation of K-3 educators. Typically, the mathematics requirements for an ECE credential are negligible; in contrast there are at least some (although often minimal) math requirements for K-3 certification. No state has credentialing requirements specifically related to teaching math at the preschool level. Moreover, colleges have a limited capacity and too few faculty who are expert in early childhood math education to provide offerings for ECE teachers, even if they were to be mandated. Currently ECE educators face diverse requirements, ranging from none to a few units of early childhood education credits taken at a two-year college, to a BA.

The field of education currently lacks the capacity to substantially improve young children's access to effective mathematics learning opportunities. Researchers who study effective teaching and learning in ECE and the early elementary grades are scarce, and few college faculty who prepare teachers are knowledgeable about math teaching.

Professional development opportunities and training are needed in early math.

Better preparation is not sufficient; teachers need ongoing support after they have begun teaching. Teachers need to be given time to participate in continuing education that is directly related to their practice. Most early childhood education programs and schools lack internal capacity, such as coaches with expertise in teaching math to young children. Such individuals could teach math directly to children across classrooms as well as support other teachers.

Research has shown that efforts to prepare preschool and K-3 teachers to be effective math teachers lead to them having better knowledge and a greater inclination to teach mathematics (Ginsburg & Ertle, 2008; Platas, 2011a,b; Sarama, DiBiase, Clements & Spitler, 2004). And preparing teachers to help parents and other caregivers promote mathematics learning at home could expand children's access to math learning opportunities.

Children need to experience programmatic coherence, especially between ECE and the early elementary grades.

There is little guidance and few support structures for early mathematical activities, and weak linkages between early childhood and elementary school mathematics. ECE activities are often not structured to build the mathematical competencies that will serve children well in kindergarten, and elementary school mathematics instruction often fails to build on what children know.

Session 4:

Policy and Advocacy

What state and federal policies are needed with regard to standards, assessment, and teacher/child development specialist credentialing? How can these policies be effectively promoted?

> **Myth #7:** Attention to mathematics detracts from opportunities to develop social-emotional and literacy skills.

Policy recommendations need to be considered in the complex context of ECE and K-3 education as they now exist. First, K-3 and ECE education typically occur within different institutions. There is some institutional diversity for K-3 (e.g., public or private schools, charter or noncharter), but even more in ECE, which takes place in Head Start programs, state preschools, family child care homes, child care centers, public and private programs, and so on. These institutions have different sources of funding and management structures, and the teaching staff have highly varying levels of training and experience. Second, turnover among teachers and caregivers, especially at the ECE level, is extremely high. Third, although instruction focused on literacy has been embraced by a large segment of the ECE community, mathematics has made few inroads into ECE instruction as demonstrated by the disparity in time spent in a typical classroom on the two domains, and there may be some resistance to it. With this context in mind, conference participants discussed what needs to be done with regard to a number of policy issues.

Key policy levers could include instituting standards for what children must learn, assessments to monitor their progress, and requirements for teachers. Standards are critically important because they create the foundation upon which all other policies and practices must be based, including student assessment, program assessment and accountability, and teacher training and credentialing.

Standards

The Common Core State Standards, developed by the National Governors Association and the Council of Chief State School Officers to apply to all grades from kindergarten through high school, have been adopted by 46 states. States are in various stages of aligning their curricula, teacher preparation and professional development to be consistent with the Common Core's generally more rigorous standards, and new assessments are being developed. Nothing comparable exists for preschool-age children. Instead, states vary greatly in standards for care and education prior to kindergarten enrollment. Head Start has recently developed its own new standards, which carry special weight because of the scale of the Head Start program. In addition, many of the ECE standards are not aligned with what will be demanded in kindergarten once the new Common Core State Standards are fully implemented. Finally, there is huge variability in how current standards are organized and delineated. Some states refer specifically to math skills, but primarily with a focus on numbers and counting; others embed mathematics in broader "cognitive-development" categories, and the number of math indicators ranges from 3 to 193.

States should examine their standards carefully to ensure continuity in mathematics for children before and after the transition from ECE to elementary school, regardless of whether these standards are developed at the state or national level. Standards should emphasize understanding rather than simply performance that could be demonstrated without real understanding. For instance, recognizing and naming shapes such as prototypically illustrated triangles, does not imply an understanding that all triangles have three angles and three sides. The National Research Council's report, *Mathematics Learning in Early Childhood,* is a useful guide in determining which mathematical ideas are important to include and in what sequence.

Formative and summative assessments should measure all critical early math concepts.

Formative assessments are conducted to help teachers understand where children need more help and guidance in early math. They may be based on one-on-one planned assessments of a single child and on less formal observations of children while they are engaged in classroom math activities. Summative assessments are tests that assess children's overall math skills or attainment of specific standards, and are typically done at the end of an academic year. The two consortia that are developing assessments to accompany the new Common Core standards — the Smarter Balanced Assessment Consortium and the Partnership for Assessment of Readiness for College and Careers (PARCC) — are developing both formative and summative assessments for mathematics in K-12. Most current assessments of young children's math understanding — formative or summative — include a very limited number of math concepts.

Program Assessments and Accountability

Quality can be assessed in terms of either what programs offer (the input) or gains children make on valued dimensions (the output). For elementary school grades, accountability is currently based almost entirely on child academic outcomes, typically their academic skills in reading and mathematics. In contrast, some states have adopted Quality Rating and Improvement Systems (QRIS)[5] to assess and then make public the level of quality (in terms of input) in child care and preschool programs. Such systems typically include both observations of the classroom (e.g., its cleanliness and safety, and the extent to which it has adequate materials; the interactions between teachers and students) and other information about the program, such as adult-child ratio. They vary substantially across states and most do not include information specific to math learning opportunities.

Quality assessments at both the ECE and K-3 levels, whether using a QRIS or other instruments, should be done regularly and should include classroom observations that document math instruction and children's other opportunities to learn math.

Teacher Training and Credentialing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All states have specific credentialing requirements for elementary school teachers to be credentialed, although in many cases the requirements for math are weak. (See Section 3). Only a few states offer a teaching credential that is specific to young children, ages 3 to 8 years. If more states offered such a credential, then more teachers of young children would have knowledge and skills specific to that developmental period. This would lead to greater continuity between ECE and the early elementary grades. Such a credential should be explicit about mathematical requirements (as well as requirements for literacy, social-emotional development, cultural sensitivity, etc.).

Short of creating a credential focused on young children, requirements related to math teaching for elementary school teachers should be strengthened and requirements for ECE teachers should be developed through existing structures such as the Child Development Associates credential. Requirements should include opportunities to learn what is known about young children's development related to mathematics and strategies for assessing understanding and teaching math to young children. New teachers should also be required to demonstrate that they have sufficient math skills themselves to teach and some of their pre-service training should include supervised student-teaching experiences.

Credentialing Institutions

More stringent pre-service requirements related to teaching math to young children will necessitate expanded offerings in institutions that provide pre-service training. States need to review the curriculum and training opportunities offered by two- and four-year colleges to ensure that students can learn to teach mathematics effectively to young children.

5 See: http://qrisnetwork.org/

Creating the Will: Spreading the Word that Math Matters

Mathematics has been neglected in educational settings for young children, but change is possible. The shift in recent years to focus on the importance of early reading has increased investment in the field and bolstered capacity among teachers and teaching institutions (McGee & Dail, 2010). It has led to changes in practice in classrooms and in homes, as parents have learned the value of reading to their children (Dickinson, 2001; Farran, Aydogan, Kang & Lipsey, 2006). That change began with research findings that demonstrated the importance of early reading and strategies that can be implemented in homes and at schools to help children develop their reading skills.

Conference participants encouraged analogous concerted efforts to bring the importance of early math learning to the attention of policy makers, educators, and the public, making sure, for example, that K-12-focused STEM initiatives be expanded to include preschoolers, and that organizations and professional development opportunities for ECE teachers routinely focus on math teaching and learning. Initiatives should emphasize that early childhood learning is not a zero-sum game: mathematics activities in the context of play can foster literacy and social-emotional development in productive ways. For example, explaining why a method for sharing cookies is fair or not fair can contribute as much to children's ability to formulate and express their reasoning clearly as it does to their mathematical development. And collaborating with peers on a mathematical game, such as Chutes and Ladders, can contribute to their ability to cooperate with other children.

Moving Forward:
Conclusions and Recommendations

Early math matters. Children who do well mathematically early on do better later on too — and not just in mathematics (Duncan et al., 2007; Grissmer, 2011; Romano et al., 2010). Early math is a natural part of development, but its development needs to be facilitated. From their very first days, infants attend to numerical and spatial patterns; with time they develop a range of mathematical ways of understanding the world that serve as a basis for further learning (see, e.g., Devlin, 2005). Children are inquisitive. They can learn to think mathematically and can learn important mathematical practices (seeing patterns, persevering, explaining), in part through play. Early mathematical activities, such as games with an underlying mathematical structure (tic-tac-toe, Chutes and Ladders, and others) can be engaging ways for children to build mathematical habits of mind, and can help prepare them for the mathematical activities they will encounter in school[6]. *All* children should be supported from the beginning in developing their innate capacity to learn math, just as they should be helped to develop their innate capacity to learn language.

6 As noted above, however, it is not safe to assume that children will pick up the mathematical ideas in games or other activities without guidance. Thus, either structured curricula or teachers trained to be well-attuned to such understandings and able to support children's learning without a curriculum are necessary.

Although mathematics is to some degree a hierarchically-organized discipline, mathematics *learning* is not necessarily linear or hierarchical. What matters most is children's readiness to engage in particular activities, and the richness of the contexts in which they engage. What adults need is sensitivity to mathematics, a more nuanced approach to the sequencing of information[7], and an understanding that there may be various profitable ways of teaching children mathematical concepts. Adults need a toolkit of activities that they can use to meet children where they are mathematically.

Relevant stakeholders need to be made aware of the importance of early childhood mathematics, and of the ways in which it can be supported and promoted.

Early mathematics learning has been neglected. Consequently, policy makers, practitioners, and parents should take action to ensure appropriate math learning opportunities for young children. The following are recommendations for supporting young children's mathematical growth.

Recommendations for Research

Basic Research: Understanding Children's Development of Mathematical Skills

- ***Study the mechanisms underlying a key finding that motivated the conference, that early mathematical proficiency predicts not only later mathematical success but also success in other academic domains.*** In particular, we need to understand how later success may be seeded by promoting early mathematical proficiency, and to what degree mathematics instruction can foster related cognitive skills (short-term memory; impulse control) that are known to contribute both to academic achievement and social skills.
- ***Develop a deeper understanding of mathematics developmental progressions, including how they vary and can be affected by instruction.***
- ***Locate and analyze existing longitudinal data sets that contain detailed assessments of early childhood mathematical competencies across a variety of sub-areas and that follow children into early elementary grades or beyond.*** The focus of this research should be to identify particular areas of learning that are most important for later learning (e.g., early patterning abilities and later algebraic reasoning) and that could be emphasized in ECE and early elementary-grade classrooms. These analyses would expand the work of Duncan et al. and others who have out of necessity relied upon the less detailed, standardized math assessments used in large national studies of child development.

Applied Research: Developing Services for Teachers and Students

- ***Gather data on the effects of teachers' beliefs and of professional development on classroom math instruction.*** Such information should help us understand the resulting effects on children's acquisition of math knowledge.
- ***Develop demonstration projects that test what young children can achieve mathematically when they are provided with culturally relevant and mathematically supportive and engaging activities.***

7 For example, although multiplication and division are typically thought of as being much more advanced than addition and subtraction, there are many contexts (e.g., sharing) in which children can multiply and divide in elegant ways before they can do some addition and subtraction problems. We can use children's innate sense of fairness to build the conceptual underpinnings of division long before those children can employ formal mathematical procedures that embody division of whole numbers.

Applied Research: Developing Measures to Tailor Instruction, Provide Accountability, and Improve Educational Quality

- ***Develop more richly textured assessments that reveal children's depth of understanding related to the critical math concepts in early childhood.*** More sensitive assessments that examine diverse pathways to competence are likely to reveal that young children from economically disadvantaged backgrounds possess more math skills than they demonstrate on current assessments. New instruments need to be developed that assess children's mastery of all of the critical math concepts in early childhood, and that tap deep understanding. Such assessments should be developmentally appropriate, both in content and form. It is *not* appropriate to subject young children to extended formal testing, but it *is* possible to engage children in activities that reveal their level of understanding of core mathematical ideas.
- ***Develop tools that teachers can use easily as formative assessments to tailor instruction for children.***
- ***Ensure that summative assessments designed for program accountability are supplemented with and aligned to formative assessments.***
- ***Develop better observational measures of teaching that can be used to assess instructional quality and improve mathematics teaching (see Farran & Hofer, in press).*** Most current observational measures of classroom quality do not include assessments of math instruction. Although observational measures of math instruction exist, either they are limited to assessing the fidelity of implementing a particular math curriculum, and thus do not assess the broader classroom context and opportunities for children to learn math, or their reliability and validity have not been fully tested.

Recommendations for Practice

- ***Create documents, curricula, and professional development materials that are guided by what we know about developmental progressions and are aligned with standards.*** They should provide rich examples of children's development of math skills, helping teachers and caregivers to "meet children where they are" and move them forward mathematically.
- ***Create problem-solving games and contexts in which children can develop mathematical knowledge and practices.*** Support should also be offered for teachers and caregivers in identifying the mathematical content of children's games and activities so that they can help children use them to think mathematically. When possible, these games and activities should reflect the cultural backgrounds of students and their families.
- ***Devote time to mathematics.*** Time should be set aside in ECE settings for intentional mathematical engagement/instruction. (This should not preclude math integrated throughout the day, including in the context of play, just as literacy activities are often distributed through the day.)
- ***Provide in-service and practice-based professional development to help current teachers develop greater knowledge of young children's mathematical thinking.***
- ***Create formal and informal structures for teacher interactions with each other and with math education experts.***
- ***Create an early childhood math teacher corps*** — a cadre of experts to serve as resources for early childhood educators.

Recommendations for Policy

- ***Create a respected commission to craft preschool common core standards in mathematics, which articulate with the Common Core State Standards for K-12 Mathematics.*** It is critically important to include individuals who have expertise in math development and learning in the standards-writing process.
- ***Create credential and college-level programs to develop experts in early childhood math teaching and learning.*** At present, there are not enough teachers and caregivers with the skills to do what is recommended in this report. Ultimately, every child care and preschool program should have resident expertise available in the form of a math teacher, consultant, or coach — just as every elementary school should. Some teacher education programs that prepare elementary teachers should provide opportunities for students to develop deep expertise in mathematics teaching to serve in such instructional leadership roles.
- ***Increase requirements related to mathematics teaching in current ECE and elementary teacher education programs.***
- ***Develop scholarships and loan forgiveness programs to attract talented people into teaching math in early childhood education programs and elementary schools.***
- ***Develop strategies to increase the number of experts in early math teaching.*** At present there is an inadequate supply of faculty and coaches prepared to help with the mathematical preparation or professional development of early mathematics teachers. Strategies for enhancing this population include scholarships or loan forgiveness programs for people preparing to teach teachers of young children, and professional development opportunities for people who are in those positions but lack expertise in the teaching of mathematics.
- ***Develop a national agenda in the U.S. Department of Education that focuses on the importance of early mathematics.*** Only this kind of national push will provide the spotlight on mathematics needed to promote increased attention to mathematical activities in early childhood education.

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